CONVECTION IN CAVITIES OF VARIABLE WALL TEMPERATURE

A. V. Lykov and B. M. Berkovskii

UDC 536.25

Nonthreshold excitation of natural convection is predicted in a horizontal cavity whose upper surface is heated in an arbitrarily nonuniform manner. From a conducting liquid in a magnetic field, it is shown that convection is not excited for an arbitrary nonuniform heating of the upper surface. The results are valid in rheological systems.

In studies of heat and mass transfer in volumes occupied by liquids and gases, the conditions for the appearance of convection are of considerable importance. Convective heat and mass transfer is known to be considerably more intense than molecular transfer, so the appearance of convection in particular cases may be either useful or undesirable, depending on whether the intention is to intensify or reduce the transfer.

1. We can determine the conditions for the appearance of convection in channels and cavities with arbitrary wall temperature. For this purpose, we will analyze the mechanical equilibrium equation $v_i = 0$ and find the class of boundary conditions for which there are no solutions. The examples of channels and cavities of planar geometry will be discussed. We assume that the gravitational force is the only mass force acting and that the physical parameters of the medium are constant. Then the equations for mechanical equilibrium are

$$-\frac{\partial P}{\partial x}+\rho g_x=0, \tag{1}$$

$$-\frac{\partial P}{\partial y}+\rho g_y=0, \tag{2}$$

$$-\frac{\partial P}{\partial z}+\rho g_z=0, \tag{3}$$

$$\frac{\partial \rho}{\partial t}=0, \tag{4}$$

$$\frac{\partial T}{\partial t}=\varkappa\nabla^2 T. \tag{5}$$

These equations should be supplemented with the equation of state. We at first assume, as is usually done in a study of free convection, that in writing the equation of state we can neglect the pressure dependence of the density and can linearize the temperature dependence of the density [1-3]. Then we find

$$\rho=\rho^*[1-\beta(T-T^*)], \tag{6}$$

where ρ^* and T^* are certain empirical constants, and $\beta=(-1/\rho)(\partial\rho/\partial T)$ is the coefficient of thermal expansion.

Let us consider the cavity to be a parallelepiped one of whose sides is parallel to the gravitational force. The boundary and initial conditions for Eqs. (1)-(6) are

$$t=0: \qquad \rho=\rho_0(x,\ y,\ z),\ T=T_0(x,\ y,\ z),$$

$$t>0: \qquad x=d \quad P=P_d(y,\ z,\ t),\ T=T_d(y,\ z,\ t),$$

Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 15, No. 6, pp. 969-974, December, 1968. Original article submitted July 5, 1968.

$$\begin{aligned} &x=-d && T=T_{-d}(y,\ z,\ t);\\ &y=h \quad P=P_h(x,\ z,\ t), && T=T_h(x,\ z,\ t),\\ &y=-h && T=T_{-h}(x,\ z,\ t);\\ &z=l \quad P=P_l(x,\ y,\ t), && T=T_l(x,\ y,\ t),\\ &z=-l && T=T_{-l}(x,\ y,\ t). \end{aligned} \tag{7}$$

We choose the y axis parallel to the gravitational force. Then $g_x = g_z = 0$ and $g_y \neq 0$ and it follows from Eqs. (1)-(4) that the pressure does not change along the x and z directions, i.e., in the plane perpendicular to the vector $\vec{g}$. Furthermore, the density should not be a function of the time $\rho \neq \rho(t)$. It also follows from Eq. (2) that the density is constant in the xz planes and thus depends only on the y coordinate: $\rho = \rho(y)$. It is assumed here that the gravitational field is constant in the xz planes, but that it may depend in an arbitrary manner on the time and the y coordinate: $g_y = g_y(y, t)$. It follows directly from this equation of state that the temperature, like the density, may not depend on the time or the x and z coordinates, and should be a function only of y: $T = T(y)$.

Taking into account these limitations on the coordinate and time dependences of the temperature, density, and pressure, we can easily find the general solution of Eqs. (1)-(6) and formulate the boundary and initial conditions under which mechanical equilibrium is possible:

$$T=T_0=T_d=T_{-d}=T_l=T_{-l}=ay+b, \tag{8}$$

$$T_h=ah+b,\ T_{-h}=-ah+b,\ a=\text{const},\ b=\text{const}, \tag{9}$$

$$\rho=\rho_0=\rho^*\left[1-\beta\left(ay+b-T^*\right)\right], \tag{10}$$

$$P=P_l=P_d=f(t)+\rho^*\int_0^y g_y(y,\ t)\left[1-\beta\left(ay+b-T^*\right)\right]dy,\quad P_h=f(t). \tag{11}$$

Mechanical equilibrium is thus possible only when: 1) the temperatures of the upper and lower surfaces of the parallelepiped are held constant over the entire surface area and are not functions of the time [Eq. (9)]; 2) the temperature of the side surfaces changes linearly and is also independent of the time [Eq. (8)]; 3) the density is constant and has the height distribution given by Eq. (10); 4) the pressure at the upper and lower surfaces of the parallelepiped has an arbitrary time dependence f(t), but the pressure is maintained in a special manner at the side surfaces [Eq. (11)]. In the case of a varying gravitational field, the pressure changes at the side surfaces of the parallelepiped correspond to the $g = g(y, t)$ dependence.

When there are any deviations from these boundary and initial conditions, there are no solutions of Eqs. (1)-(6); accordingly there is no mechanical equilibrium of the liquid, so convection arises in the cavity. Convection may arise in the cavity when either the temperature boundary conditions (8) and (9) or the pressure boundary conditions are disrupted. We are interested primarily in convection excitation by means of heating. We will discuss this question in detail.

Our particular interest is in the excitation of free convection in a cavity filled with a gas or a liquid and whose upper surface is heated. As we just explained, it is sufficient here either to produce a nonuniform temperature distribution over the upper surface or to give this temperature a time dependence. Convection should arise when there are arbitrarily small deviations from a uniform temperature or from a constant temperature. Experiments should be carried out to check this possibility of exciting free convection by means of heating from above. This can apparently be done most easily by producing a stepped, or approximately stepped, temperature distribution over the upper surface of the parallelepiped filled with the liquid or gas:

$$T_h=\begin{cases} T_{h0} & -d\leqslant x\leqslant 0\\ T_{h0}+\Delta T & 0<x\leqslant d, \end{cases}\quad z,\ t-\text{arbitrary}$$

or

$$T_h=\begin{cases} T_{h0} & 0\leqslant t,\\ T_{h0}+\Delta T & t>0, \end{cases}\quad x,\ z-\text{arbitrary},$$

and by maintaining the lower surface at $T_{-h} = T_{h0}$. Although convection is predicted theoretically at arbitrarily small ΔT, the possible existence of a critical value ΔT_{cr} should be noted during the experiments, and the changes in the structure of the convective flow with changing ΔT should be followed.

The possibility of exciting free convection in a strictly horizontal cavity by means of nonuniform heating from above should not be confused with free convection in a sloping channel whose upper wall is at a higher temperature [3]. In this case, the convection, which, incidentally, is observed at a constant wall temperature, is due to the noncollinearity of the sides of the channel with the gravitational vector; this convection would not arise in a horizontal channel. Both possibilities for the appearance of convection make a mechanical equilibrium in a cavity filled with a liquid or gas an exceptional phenomenon. Under actual conditions, one must generally expect convection. Accordingly, the large discrepancies between the experimental thermal and other physical characteristics of liquids and gases obtained by various authors [5-7, 11, 12] are understandable, and one recognizes the necessity for treating experimental results on the basis of a heat conduction equation incorporating convective terms.

The considerable range of critical Rayleigh numbers [8-10] which have been observed experimentally can also be attributed to the appearance of convection because the surfaces of the horizontal planar layers are not isothermal.

Let us now see what changes are caused in these results by replacement of equation of state (6) by a more accurate one. Let us consider, e.g., an ideal gas in which the equation of state is exactly

$$P = R\rho T.$$

It is difficult to see that the steady-state results will be essentially the same. In particular, to excite convection by heating from above, it is, as before, sufficient to heat the horizontal surfaces in a nonuniform manner. The temperature distribution at the side surfaces remains the same, but the pressure and density distributions are different:

$$\rho \simeq \exp\left[-\int \frac{aR - g(y)}{R(ay+b)}\,dy\right],$$
$$P \simeq (ay+b)\exp\left[-\int \frac{aR - g(y)}{R(ay+b)}\,dy\right].$$

New results are found under transient conditions. For example, there is a mechanical equilibrium when there is a certain agreement between the time dependences of the pressure, temperature, and gravitational acceleration. It is not difficult to see that this is possible when $T(t) \sim P(t) \sim g(t) \sim e^{\alpha t}$. In this case, the solution of the problem requires further study.

2. Let us determine the conditions for the appearance of convection in a cavity with nonisothermal surfaces when this cavity is filled with a conducting liquid (or gas) and is placed in a magnetic field. We assume the cavity to be a parallelepiped and assume that the only nonvanishing component of the magnetic field is directed parallel to the gravitational acceleration, which is also parallel to one side of the parallelepiped and y axis. As in the case of nonconducting liquids, we find a class of boundary conditions under which solutions of the static equation do not exist – in this case, the equations of magnetohydrostatics. We assume the magnetic field is uniform along the z axis; then the magnetohydrostatic equations become [4]

$$\begin{gathered}
H_x = H_z \equiv 0, \quad H_y = H_y(x, t), \\
\frac{\partial H_y}{\partial t} = \frac{c^2}{4\pi\sigma}\frac{\partial^2 H_y}{\partial x^2}, \\
0 = -\frac{\partial}{\partial x}\left(P + \frac{H_y^2}{8\pi}\right), \\
0 = -\frac{\partial}{\partial y}\left(P + \frac{H_y^2}{8\pi}\right) + \rho g_y, \\
0 = -\frac{\partial}{\partial z}\left(P + \frac{H_y^2}{8\pi}\right), \\
\frac{\partial \rho}{\partial t} = 0, \\
\frac{\partial T}{\partial t} = \varkappa\Delta T + \frac{c^2}{16\pi^2\sigma\rho^* c_p}\left(\frac{\partial H_y}{\partial x}\right)^2,
\end{gathered} \tag{12}$$

where σ is the conductivity and c is the electrodynamic constant. We use the equation of state as given in (6). The boundary and initial conditions (7) must be supplemented by conditions for the magnetic field components:

$$\begin{aligned} & t=0 \quad H_y = H_{y0}(x), \\ t>0 \quad & x=l \quad H_{yl} = H_{yl}(t), \\ & x=-l \quad H_y = H_{y(-l)}(t). \end{aligned}$$

Without going into a detailed analysis of system (12), we state the general solution and boundary conditions for a mechanical equilibrium:

$$H_y = H_{y0} = Ax + B, \; H_{yl} = Al + B, \; H_{y(-l)} = -Al + B,$$

$$T = T_0 = T_d = T_{-d} = T_l = T_{-l} = \alpha y^2 + ay + b,$$

$$T_h = \alpha h^2 + ah + b, \; T_{-h} = \alpha h^2 - ah + b,$$

$$\rho = \rho_0 = \rho^* [1 - \beta (\alpha y^2 + ay + b - T^*)],$$

$$P = f(t) + \int \rho(y) g(y, t)\, dy - \frac{(Ax+B)^2}{8\pi},$$

$$\alpha = \frac{Ac^2}{32\pi^2 \sigma \rho^* c_p \varkappa}, \quad A, B, a, b - \text{const.}$$

It was assumed for this solution that the gravitational field was constant in the xz planes.

Any deviation from these conditions should cause macroscopic motion.

As before, nonisothermal horizontal surfaces (upper or lower) cause convection. Macroscopic motion in the cavity is also excited when the magnetic field or temperature is other than constant in time. Interestingly, mechanical equilibrium cannot occur when the magnetic field has a nonlinear dependence on the x coordinate.

Using the equation of state for an ideal gas, we find a completely different result, even for the steady-state problem. In a magnetic field which varies linearly along the x axis, the horizontal surfaces may be nonisothermal at mechanical equilibrium if the pressure along these surfaces changes in a specified manner:

$$H = Ax + B.$$

$$T = [8\pi \Pi(y) - (Ax+B)^2] (8\pi\rho)^{-1},$$

$$P = \Pi(y) - \frac{(Ax+B)^2}{8\pi},$$

$$\rho = \frac{d\Pi}{dy} \frac{1}{g},$$

$$\Pi = (Dy + E) \left\{ K + Ly - \left(\frac{a^2 E}{8\pi} + \frac{\alpha}{2} \right) y^2 - \frac{a^2 D}{24\pi} y^3 \right\},$$

$$D, E, K, L = \text{const.}$$

The choice of equation of state, as we will see, plays an important role with respect to the conditions for convection excitation; this choice will be discussed elsewhere in more detail.

We note in conclusion that these results regarding the excitation of free convection as a result of nonisothermal surfaces can be converted without difficulty to excitation due to nonisobaric conditions; these results hold for Newtonian liquids (or gases) and in rheological systems.

LITERATURE CITED

1. A. V. Lykov and Yu. A. Mikhailov, Theory of Heat and Mass Transfer [in Russian], Gosénergoizdat, Moscow (1963).
2. L. D. Landau and E. M. Livshits, Continuum Mechanics [in Russian], GITTL, Moscow (1954).
3. G. A. Ostroumov, Free Convection under Interior-Problem Conditions [in Russian], GITTL, Moscow –Leningrad (1952).

4. L. D. Landau and E. M. Livshits, Electrodynamics of Continuous Media [in Russian], GIFML, Moscow (1959).
5. L. B. Filippov, Vestnik MGU, Fizika, No. 6 (1954).
6. L. B. Filippov, Vestnik MGU, Fizika, No. 3, 61 (1960).
7. Yu. L. Rastorguev and Yu. A. Ganiev, Zh. Fiz. Khim., 41, 2901 (1967).
8. P. A. Norden and A. G. Usmanov, Heat and Mass Transfer, Vol. 1: Heat and Mass Transfer during the Interaction of Objects with Flowing Liquids and Gases [in Russian], Energiya, Moscow (1968).
9. De Graaf and Van der Held, Appl. Sci. Res., A3, 393 (1954).
10. K. Chandra, Proc. Roy. Soc., A164, 231 (1938).
11. H. L. Sheng, H. Eyring, and W. J. Davis, J. Phys. Chem., 68, No. 10, 3017 (1964).
12. N. Bennard, P. Johanin, and B. C. Vodar, R. Acad. Sci., 261, 67 (1965).